Proposal of A Humanlike Two-Joint Link Mechanism Provided with the Bi-articular and the Mono-articular Actuators

Part 2 : Trajectory Control : Contact Task was dissolved

FUJIKAWA Tomohiko*, OSHIMA Toru** and KUMAMOTO Minayori***

*Department of Electronic Control Engineering, Toyama National College of Maritime Technology
Shiminoato, Toyama, 933-0293 Japan
**Faculty of Engineering, Toyama Prefectural University
Kosugi, Toyama, 939-0398 Japan
*** Laboratories of Image Information Science and Technology
Hongo 5-26-4, Bunkyo, Tokyo,113-0033 Japan

This paper describes the trajectory control of a humanlike two-joint link mechanism to dissolve the contact task problem. Mechanical properties of the endpoints of the two-joint link mechanisms operated with the muscle coordinate system consisting of three pairs of antagonistic actuators with bi-articular actuators and with the joint coordinate system consisting of two pairs of antagonistic actuators without bi-articular actuators, were comparatively examined theoretically and experimentally in terms of robotics. The results obtained here demonstrate that the muscle coordinate system contributes to control the output force, the elastic ellipse, and the trajectory exerted at the endpoint, and the two-joint link mechanism provided with the three pairs of antagonistic actuators could dissolve the contact task.

1. INTRODUCTION

In general, a conventional robot arm has the joint coordinate system constructed with an equal number of actuators to the number of its joints, whereas a human extremity has the muscle coordinate system constructed with three pairs of the antagonistic muscles consisting of one antagonistic pair of bi-articular muscles and two antagonistic pairs of mono-articular muscles. With regard to control properties resulting from the existence of the antagonistic bi-articular muscles, N. Hogan(1),(2) has reported that an antagonistic pair of bi-articular muscles could contribute to the impedance control of the endpoint of the extremity, and J. McIntyre and E. Bizzi(3) have proposed an Equilibrium point control model wherein an antagonistic pair of bi-articular muscles could contribute to positional control of the endpoint. Recently we have reported that the existence of an antagonistic pair of bi-articular muscles could positively contribute to stiffness control and trajectory control of the endpoint of the extremity.(4),(5) Furthermore, on the basis of results obtained from electromyographic kinesiology analysis of the human upper extremity and control engineering analysis carried out on the two-joint link model consisting of one antagonistic pair of bi-articular actuators and two antagonistic pairs of mono-articular actuators, we have reported that the antagonistic pairs of bi- and mono-articular muscles in the human upper extremity demonstrated perfectly coordinating activity patterns, and that the patterns could contribute to output force control and output force direction control at the endpoint of the extremity, whereby we proposed Coordination control model.(6)

In this part of the present studies, the trajectory control properties of the two-joint link mechanisms constructed with the joint coordinate system and with the muscle coordinate system were comparatively examined in terms of robotics, and simple robotic experiments were designed to show how the humanlike two-joint link mechanism could dissolve the contact task, which has never been dissolved in the conventional, even in the most sophisticated robot arm mechanism.

2. MUSCLE COORDINATE SYSTEM

In a two-joint link model of the muscle coordinate system consisting of one pair of antagonistic bi-articular actuators and two pairs of antagonistic mono-articular actuators as shown in Fig.1, all actuators are assumed to have the same characteristics.

A muscle model used in the present mechanical analyses is an elastic model. Output forces of the muscle model, $F_m$, are given as follows:

$$f_m = u_m \cdot (k_m \cdot u_m \cdot z_m),$$

where; $k_m$ and $u_m$: muscular elastic coefficient, $u_m$ and $u_m$:
muscular contractile force, }x_1\text{ and }x_2\text{; muscular contraction length, }i\text{; muscle number. Joint torques (}T_1\text{, }T_2\text{) in Fig. 1 are given as follows:}

\[ T_1 = (r_1 - f_1) \cdot \phi_1 + (r_2 - f_2) \cdot \phi_2 \]
\[ T_2 = (r_2 - f_2) \cdot \phi_2 + (r_1 - f_1) \cdot \phi_1 \]

(2)

where; }r_1\text{ and }r_2\text{; radii of joint pulleys at joints }J_1\text{ and }J_2\text{, respectively. Generally, in the task coordinate system, the relationships between coordinates of the endpoint }E, \{(x, y)\text{, and the joint angles of } \theta_1\text{ and } \theta_2\text{ shown in Fig. 1 are given as follows:}

\[ x = x_1 \cdot \cos \theta_1 + x_2 \cdot \cos (\theta_1 + \theta_2) \]
\[ y = y_1 \cdot \sin \theta_1 + y_2 \cdot \sin (\theta_1 + \theta_2) \]

(3)

where; }l_1\text{; length of the segment between joints }J_1\text{ and }J_2\text{, }l_2\text{; length of the segment between joint }J_1\text{ and endpoint }E\text{. The relation between joint torques (}T_1\text{, }T_2\text{) and }X\text{- }Y\text{-}Z\text{ components of the force exerted at the endpoint }E, \{(F_x, F_y, F_z)\text{ is given by:}

\[
\begin{bmatrix}
T_1 \\
T_2 \\
F_x \\
F_y \\
F_z
\end{bmatrix} =
\begin{bmatrix}
\alpha & -\beta & 0 \\
\gamma & 0 & -\delta \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
F_z
\end{bmatrix}
\]

(4)

\[ \alpha = l_1 \cdot \sin \theta_1 + l_2 \cdot \sin (\theta_1 + \theta_2), \gamma = l_2 \cdot \sin (\theta_1 + \theta_2), \]
\[ \beta = l_1 \cdot \cos \theta_1 + l_2 \cdot \cos (\theta_1 + \theta_2), \delta = l_2 \cdot \cos (\theta_1 + \theta_2). \]

The relations between six muscular contracting lengths (}x_0, x_0, x_0, x_0, x_0, x_0\text{ and the joint angles of } \theta_1\text{ and } \theta_2\text{ are given as follows:}

\[ x_1 = r_1 \cdot \theta_1, x_2 = r_2 \cdot \theta_2 \]
\[ x_3 = r_1 \cdot \theta_1 + r_2 \cdot \theta_2, x_4 = r_1 \cdot \theta_1 - r_2 \cdot \theta_2, x_5 = r_1 \cdot \theta_1 - r_2 \cdot \theta_2. \]

(5)

From Eqs. (1)–(5), the relation between very small changes in coordinates of the endpoint }E, \{(\Delta x, \Delta y)\text{ and very small changes in the }X\text{- }Y\text{-}Z\text{ components of the force exerted at the endpoint }E, \{(\Delta F_x, \Delta F_y, \Delta F_z)\text{ is derived as follows:}

\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta F_x \\
\Delta F_y \\
\Delta F_z
\end{bmatrix} =
\begin{bmatrix}
K_{11} & K_{12} & \Delta F_1 \\
K_{21} & K_{22} & \Delta F_2 \\
K_{31} & K_{32} & \Delta F_3 \\
K_{41} & K_{42} & \Delta F_4 \\
K_{51} & K_{52} & \Delta F_5
\end{bmatrix}
\]

(6)

\[ K_1 = (u_1 - k_1 + u_1) \cdot r_1, K_2 = (u_2 - k_2 + u_2) \cdot r_2, \]
\[ K_3 = (u_3 - k_3 + u_3) \cdot r_3, K_4 = (u_4 - k_4 + u_4) \cdot r_4, \]
\[ K_5 = (u_5 - k_5 + u_5) \cdot r_5, K_6 = (u_6 - k_6 + u_6) \cdot r_6. \]

The potential energy }E_p\text{ at the endpoint }E\text{ is given as follows:

\[
E_p = K_1 + K_2 + K_3 + K_4 + K_5 + K_6 + \Delta x^2 K_1 + \Delta y^2 K_2.
\]

(7)

An elastic ellipse can be derived from the relation between the potential energy }E_p\text{ and the muscular elastic coefficient, and an elastic coefficient }k_2\text{ at endpoint }E\text{ in any direction can be indicated on the ellipse as shown in Fig. 2. The elastic ellipse consists of three parameters; the lengths of the long and short axes (}b, a\text{) and inclination (}\theta\text{). Under the assumption that the lengths of the links and radii of the pulleys are held constant, }l_1 = l_2 = r_1 = r_2\text{, and }K_2 = K_4\text{, the relation between the three parameters of the elastic ellipse (}a, b\text{, }\theta\text{) and elasticities of three pairs of antagonistic muscles (}k_1, k_2, k_3\text{) is derived as follows:}

\[
\begin{bmatrix}
K_1 \\
K_2 \\
K_3
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & -\cos \phi & \sin \phi & 0
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 \\
C_1
\end{bmatrix}
\]

(8)

\[ \xi \sin \phi + \sin \phi, \xi \cos \phi + \cos \phi, \phi = \theta_2 - \theta_1, \mu = \theta_2 - \theta_1. \]

From Eq. (8), it follows that the elastic ellipse exerted at the endpoint }E\text{ can be controlled by the sum of muscular contractile forces of any pair of antagonistic muscles. Three pairs of antagonistic muscles, including mono- and bi-articular muscles, are sufficient for adjusting the three parameters of the elastic ellipse. Therefore, shape and inclination of the elastic ellipse can be determined independently.

In the muscle coordinate system model equipped with 3 pairs 6 actuators, from Eq. (8), if all actuators have supposedly the same elasticity (}K_1 = K_2 = K_3\text{), three parameters of the elastic ellipse exerted at the endpoint }E, \{(A_1, B_1, C_1)\text{ are given as follows:}

Fig. 2 Elastic ellipse

Fig. 3 Changes in the elastic ellipse with the postural changes (muscle coordinate system)
as follows:
\[ A_p = 3/(2R_k \sin^2(\theta_t/2)), \]
\[ B_p = 1/(2R_k \cos^2(\theta_t/2)), \]
\[ \theta_t = (2 \theta_t + \theta_2)/2. \]
(9)

From Eq. (9), the changes in shape of the elastic ellipse with changes in the postural condition are shown in Fig. 3. As shown in Fig. 3, the major axes of the elastic ellipses exerted at the endpoint E are always passing throw the endpoint E and the first joint J1, along axis X, regardless of changes in posture.

3. JOINT COORDINATE SYSTEM

In a two-joint model of the joint coordinate system consisting of two pairs of antagonistic mono-articular actuators, without the bi-articular actuators (f1 and e3), as shown in Fig. 4, all actuators are assumed to have the same characteristics. As was mentioned previously, a muscle model used in the present mechanical analyses is an elastic model. Output forces of the muscle models, \( F_m \), are given as follows:
\[ f_m = u_0 - k_m x_m, \]
\[ f_m = u_0 - k_m x_m, \]
\[ m = 1, 2. \]
(10)

where: \( k_0 \) and \( k_m \): muscular elastic coefficient, \( u_0 \) and \( u_0 \): muscular contractile force, \( x_m \) and \( x_m \): muscular contraction length, \( m \): muscle number. Joint torques \( T_1 \) and \( T_2 \) in Fig. 4 are given as follows:
\[ T_1 = (f_1 - f_0) r_1, \]
\[ T_2 = (f_2 - f_0) r_2. \]
(11)

where: \( r_1 \) and \( r_2 \): radii of joint pulleys at joints 1 and 2, respectively. In the task coordinate system, the relationships between coordinates of endpoint \( E \), \( (x, y) \), and the joint angles of \( \theta_1 \) and \( \theta_2 \) shown in Fig. 4 are given by Eq. (3). The relation between joint torques \( T_1 \) and \( T_2 \) and X-Y components of the force exerted at the endpoint \( E \), \( (F_x, F_y) \), is given by Eq. (4). The relations between four muscular contracting lengths \( x_k, x_k \) and the joint angles of \( \theta_1 \) and \( \theta_2 \) are given as follows:
\[ x_{k1} = r_1 \theta_1, \]
\[ x_{k2} = r_2 \theta_2, \]
\[ x_{k2} = x_{k2} = -r_2 \theta_2, \]
(12)

From Eqs. (3), (4), (9)–(11), the relation between very small changes in coordinates of the endpoint \( E \), \( \Delta x, \Delta y \), and very small changes in the X-Y components of the force exerted at the endpoint \( E \), \( \Delta F_x, \Delta F_y \), is derived as follows:
\[ \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \Delta F_x \\ \Delta F_y \end{bmatrix}, \]
(13)

\[ K_{11} = (u_0 - k_1 x_1+k_0) r_1^2 , \]
\[ K_{12} = (u_0 - k_2 x_2+k_0) r_2^2 , \]
\[ K_{21} = -k_1 x_1 , \]
\[ K_{22} = -k_2 x_2 . \]

The potential energy \( E_p \) at the endpoint \( E \) is given by Eq. (7). An elastic ellipse can be derived from the relation between the potential energy \( E_p \) and the muscular elastic coefficient, and an elastic coefficient \( (k_e) \) at endpoint \( E \) in any direction can be indicated on the ellipse as shown in Fig. 2. The elastic ellipse consists of three parameters \( (A_p, B_p, \theta_e) \). Under the assumption that the lengths of the links and radii of the pulleys are held constant, \( m_1 = m_2, r_1 = r_2, \) the relation between the three parameters of the elastic ellipse \( (A_p, B_p, \theta_e) \) and elasticities of two pairs of antagonistic muscles \( (k_1, k_2) \), is derived as follows:

\[ K_1 = (A_p \sin^2 \theta_e + B_p \cos^2 \theta_e)^2 - (A_p \cos^2 \theta_e + B_p \sin^2 \theta_e)^2, \]
\[ A_p = B_p = \frac{\beta \gamma - \alpha^2}{\alpha^2 \beta^2}, \]
(14)

\[ \theta_e = (\sin^{-1} \rho \tan^{-1} \sigma)/2, \]
\[ \alpha = (\alpha \beta \rho)/(\alpha \delta + \beta \gamma), \]
\[ \beta = (A + B)(\alpha \rho \beta)^2, \]
\[ \rho = (A - B)^2 ((\alpha \rho \beta)^2 + (\alpha \delta + \beta \gamma)^2). \]

From Eq. (14), it follows that the elastic ellipse exerted at the endpoint \( E \) can be controlled by the sum of muscular contractile forces of any pair of antagonistic muscles. Two pairs of antagonistic muscles without bi-articular muscles are not sufficient for adjusting the three parameters of the elastic ellipse. Therefore, shape and inclination of the elastic ellipse can not be determined independently.

In the joint coordinate system model equipped with only

![Fig. 4 Two-joint model of the joint coordinate system](image1)

![Fig. 5 Changes in the elastic ellipse with the postural changes (joint coordinate system)](image2)
2 pairs 4 actuators, from Eq. (14), if all actuators have supposedly the same elasticity \( K = K_1 = K_2 \), three parameters of the elastic ellipse exerted at the endpoint E \((A_x, B_x, \theta_x)\) are given as follows:

\[
A_x = \frac{2 \sin 2\theta_x}{K (2\alpha + \beta + \gamma + \delta) \sin 2\theta_x + 2(2\alpha \beta + \gamma \delta)},
\]

\[
B_x = \frac{2 \sin 2\theta_x}{K (2\alpha + \beta + \gamma + \delta) \sin 2\theta_x - 2(2\alpha \beta + \gamma \delta)},
\]

\[
\theta_x = \frac{1}{2} \tan^{-1} \left( \frac{2(2\alpha \beta + \gamma \delta)}{-2\alpha + \beta - \gamma + \delta} \right).
\]

(15)

From Eq. (15), the changes in shape of the elastic ellipse with changes in the postural condition are shown in Fig. 5. As shown in Fig. 5, inclination of the elastic ellipses exerted at the endpoint E changes with changes in posture, and the major axes of the elastic ellipses never coincided with axis X.

4. Trajectory and Output Force

In the elastic ellipse of Fig. 6, the elastic coefficients \( K_x \) on the X' and Y' axes are 2A_x and 2B_x, respectively. When a output force is exerted at the endpoint E in the direction of \( \Theta_x \) at the point P on the elastic ellipse, relation between very small changes in the output force \( \Delta F_x \) and very small changes in coordinates \( \Delta x', \Delta y' \) of displacement \( \Delta L' \) of the endpoint E is derived as follows:

\[
\begin{bmatrix}
\Delta x' \\
\Delta y'
\end{bmatrix} = \begin{bmatrix}
\frac{\cos \Theta_x}{2A_x} \\
\frac{\sin \Theta_x}{2B_x}
\end{bmatrix} \Delta F_x.
\]

(16)

Direction \( \Theta_x \) of very small changes in coordinates \( \Delta x', \Delta y' \) of the displacement of the endpoint E, or the trajectory of the endpoint E, would be:

\[
\tan \Theta_x = \frac{\Delta y'}{\Delta x'} = \frac{A_x}{B_x} \tan \theta_x.
\]

(17)

From Eq. (17), the trajectory of the endpoint E is expressed by the normal direction to the tangent at the point where the output force direction intersect the elastic ellipse. If the shape of elastic ellipse becomes a circle, or if the output force direction coincides with the X' or Y' axis, the output force direction could coincide with the trajectory.

The shape of ellipse can be determined by the sum of contractile forces of any antagonistic pair of muscles, and the output force can be determined independently by the difference between contractile forces of any antagonistic pair of muscles; therefore, in the muscle coordinate system the output force direction can be made to coincide with the trajectory.

5. Experimental Analysis

5.1 Experimental Arm Model

Two joint link models provided with six springs \( (f_1, e_1, f_2, e_2, f_3, e_3) \) to serve as the muscle coordinate system (a muscle coordinate system model) and with four springs \( (f_1, e_1, f_2, e_2) \) to serve as the joint coordinate system (a joint coordinate system model) were constructed in a horizontal plane, as shown in Fig. 7, for experimental analysis. All springs used have the same characteristics (elastic coefficient: 2.16N/mm, maximum contractile force: 200.3N), and were installed on the two-joint link mechanism (length: 400mm each) via spur gear, sprockets (radius: 47.8mm) and chains. Springs \( f_1 \) and \( e_1 \) drive joint \( J_1 \), and springs \( f_2 \) and \( e_2 \) drive joint \( J_2 \) to serve as antagonistic mono-articular muscles. Springs \( f_3 \) and \( e_3 \) drive both joints \( J_1 \) and \( J_2 \) simultaneously to serve as antagonistic bi-articular muscles. A postural condition was determined by the joint angles \( (\theta_1, \theta_2) \). A output force \( (F, \Theta) \) was measured by a force detector installed at the endpoint E.

5.2 Elastic Ellipse

Changes in elastic ellipse shaped by three parameters \( (A_x, B_x, \theta_x) \) with changes in experimental postural condition, were calculated by relation between small changes in coordinates of displacement of the endpoint E and small changes in the X-Y components of the output force exerted at the endpoint E. Experimental postural conditions of \( \theta_1 \) and \( \theta_2 \) were 60° and 60°, 45° and 90°, and 30° and 120°.
respectively. The results obtained with the muscle coordinate system model and with the joint coordinate system model are shown in Fig. 8 and 9, respectively. In Fig. 8, the major axes of the elastic ellipses exerted at the endpoint E always coincided with axis X. On the other hand, in Fig. 9, the major axes of the elastic ellipses exerted at the endpoint E never coincided with axis X, and the major axes changed one by one with changes in postural condition. The experimental results obtained here demonstrated exactly the same tendencies as the results obtained by the theoretical analysis with both the muscle and joint coordinate system models.

5.3 OUTPUT FORCE DIRECTION Changes in the output force direction exerted at the endpoint E with changes in displacement of the endpoint E were measured using the force detector installed at the endpoint E. The initial position of the endpoint E was set at the fully extended posture, $\theta_1=90^\circ$ and $\theta_2=0^\circ$, and the endpoint E was moved toward joint J along axis X until $\Delta \theta$ component of displacement of the endpoint E reached about 400m. Relations between displacement of the endpoint E ($x$) along axis X and the output force direction ($\theta_2$) are shown in Fig. 10. In Fig. 10, in the muscle coordinate system model, the output force direction did not show any change and did coincide with axis X, whereas, in the joint coordinate system model, the output force direction shifted more from axis X with increase in $\Delta \theta$ component of the displacement. The experimental results obtained here demonstrated almost the same tendencies as the results obtained by the theoretical analysis with both the muscle and joint coordinate system models.

5.4 TRAJECTORY DURING CONTACT TASK Trajectories...
of the endpoint E of the two-joint link model were recorded during a contact task, in which an acrylic resin plate was moved toward joint J1 keeping right angle against axis X (Fig. 11). The initial posture of the model was set at the fully extended position, $\theta_1=90^\circ$ and $\theta_2=0^\circ$, and the acrylic resin plate was placed to contact to the endpoint E at right angle to axis X. A negligible frictional resistance between the endpoint E and the acrylic resin plate will supposedly exist. In the muscle coordinate system model, the trajectory of the endpoint E perfectly coincided with axis X, as is clearly shown in Fig. 11 (a), whereas, in the joint coordinate system model, the trajectory of the endpoint E greatly shifted from axis X, as shown in Fig. 11 (b).

5. CONCLUSION

The result obtained in this part 3 of the present studies, the two-joint link mechanism operated with muscle coordinate system consisting of the three pairs of antagonist actuators including the bi-articular actuators could demonstrate the real human-like trajectory control properties and could dissolve the contact task. Whereas the two-joint link mechanism operated with the joint coordinate system consisting of two pairs of the mono-articular actuators without the bi-articular actuators could not dissolve the contact task.

6. REFERENCE


![Fig.10 Output force direction](image)

- muscles coordinate system (experimental value)
- muscles coordinate system (theoretical value)
- joint coordinate system (experimental value)
- joint coordinate system (theoretical value)

$s_1$: postural condition 1 $(\theta_1=60^\circ, \theta_2=60^\circ)$
$s_2$: postural condition 2 $(\theta_1=45^\circ, \theta_2=90^\circ)$
$s_3$: postural condition 3 $(\theta_1=30^\circ, \theta_2=120^\circ)$

![Fig.11 Trajectory of the endpoint during contact task](image)

Ac: Acrylic resin plate